FORMULATION OF MODIFIED KDV EQUATION WITH SECULAR TERM IN A MULTI-ION PLASMA WITH SUPER-THERMAL ELECTRONS

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Abstract
We have done a theoretical modeling of ion acoustic solitary waves in a four-component cometary plasma consisting cometary electron, positively charged oxygen ions of cometary origin, negatively charged oxygen ions of cometary origin and solar wind hydrogen ions. The first order K-dV equation and K-dV equation with secular term have derived by method of reductive perturbation.For numerical analysis, the parameters observed in the coma of comet Halley are used. The electron component is described by superthermal kappa distribution and the ion components are described by Maxwellian distribution.
Keywords: Solitary waves, Korteweg de-Vries (KdV) equation, secular term, multi-ion plasma, reductive perturbation.

Introduction

One of the most fundamental of oscillations in a plasma are low-frequency electrostatic or longitudinal ion density waves. The ions provide the inertia with the electrons as the source of the restoring force, in the long-wavelength limit. Ion-acoustic waves also exhibit strong nonlinear properties. In general a cometary environment contains new born hydrogen and heavier ions, with relative densities that depends on the distance from the nucleus. The Giotto's observations of the inner coma of comet Halley showed that, in addition to the usual thermal electrons and ions, fast cometary pickup ions, a new component, namely, negatively charged cometary ions was present. These negative ions were observed in three broad mass peaks at 7–19, 22–65, and 85–110amu with O^- being identified unambiguously [Chaizy et al., 1991]. Ion-acoustic waves in the frequency range of 1.0–1.5 kHz were observed by the ICE spacecraft sent to observe the comet Giacobini-Zinner and ion sound waves, with a frequency slightly less than 1 kHz, were detected by the spacecraft Sakigake which observed comet Halley [Brincaand Tsurutani, 1987, 1989, 1990; Brinca et al., 1989].

The K-dV equation, a non-linear partial differential equation whose solutions can be exactly and precisely specified, is particularly notable as the prototypical example of an exactly solvable model [Martin, 2011; Dauxois and M. Peyrard, 2006]. The history of the K-dV equation began with experiments by John Scott Russell in 1834 and was followed by theoretical investigations by Lord Rayleigh and Joseph Boussinesq around 1870. Finally, Korteweg and De Vries in 1895 did it [Philip Drazin, 1991].

The concept of solitary wave or soliton, which is a product of nonlinear wave theory, was introduced in Physics as an extensive search for electromagnetic solitary wave. It is because of their very interesting and promising features, namely their consistent shape, their constant velocity, and their ability to interact cleanly.

Theoretical Model

Cometary plasma is a true multi-ion plasma consisting of hydrogen ions and electrons of solar origin, along with hydrogen, oxygen, and associated photoelectrons of cometary origin formed by the dissociation of water molecules. The definite identification of negatively charged oxygen ions (O^-) permits one to model cometary plasma as a pair ion plasma. Many of the above ions can also be modelled as ion pairs. While the above plasma composition applies to Halley's Comet, heavy ions have also been observed at other comets. Based on studies, cometary plasma system can be modelled as multi-ion (four-component) cometary plasma consisting of one electron component, hydrogen ions, and pair ions. Both kappa and Maxwell distributions are used because of their importance in space plasmas and the observation of nonlinear events at Halley's comet.

We consider the streaming of one plasma (solar wind) through a target (the cometary plasma) with separate quasi-neutrality conditions for both plasmas.

Four component cometary plasma satisfies the quasi-neutrality condition $n_{ce} + n_{0-} = n_{sH} + n_{0+}$.

Here indices sH, +, and − denote solar hydrogen ions and positively and negatively charged pair ions, respectively. The streaming components are hydrogen and electrons of solar origin; these describe the solar wind, which is assumed to be streaming in a direction parallel to the magnetic field. Since observations of space plasmas show superthermal tails, the solar electrons can be modelled by a kappa distribution. The target cometary plasma consists of electrons and pair ions.

Basic Equations

The kappa distribution is given by,

$For \quad n_e$

$$n_{he} = \left(1 - \frac{T\phi}{\kappa - 3/2}\right)^{-(\kappa - 1/2)} \qquad \cdots\cdots(1a)$$

$For \quad n_{0+}$

$$\frac{\partial n_{o+}}{\partial t} + \frac{\partial}{\partial x}(n_{o+}u_+) = 0 \qquad \cdots\cdots(1b)$$

$$\frac{\partial u_+}{\partial t} + u_+\frac{\partial u_+}{\partial x} + \frac{\partial \phi}{\partial x} = 0 \qquad \cdots\cdots(1c)$$

$For \quad n_{0-}$

$$\frac{\partial n_{o-}}{\partial t} + \frac{\partial}{\partial x}(n_{o-}u_-) = 0 \qquad \cdots\cdots(1d)$$

$$\frac{\partial u_-}{\partial t} + u_-\frac{\partial u_-}{\partial x} - q\frac{\partial \phi}{\partial x} = 0 \qquad \cdots\cdots(1e)$$

$For \quad n_H$

$$\frac{\partial n_H}{\partial t} + \frac{\partial}{\partial x}(n_H u_H) = 0 \qquad \cdots\cdots(1f)$$

$$\frac{\partial u_H}{\partial t} + u_H\frac{\partial u_H}{\partial x} + r\frac{\partial \phi}{\partial x} = 0 \qquad \cdots\cdots(1g)$$

Instead of using the independent variables x and t, we use the following dependent variables

$$\xi = \varepsilon^{1/2}(x - V_0 t) \quad ; \quad \tau = \varepsilon^{3/2} t \qquad \cdots\cdots\cdots(2)$$

It is a kind of symmetry group of the K-dV equation which involves going to a frame of reference moving with a constant velocity V_0 with respect to the fixed frame, i.e., V_0 is the phase velocity of the wave, which is to be determined later, and ε is a smallness parameter.

Using expression (2) in (1) we get the coupled set of equations

$$n_e = \left[1 - \frac{T\phi}{\kappa - 3/2}\right]^{-(\kappa - 1/2)} \qquad \cdots\cdots(3a)$$

$$\varepsilon^{3/2}\frac{\partial n_{o+}}{\partial \tau} - \varepsilon^{1/2}\frac{\partial n_{o+}}{\partial \xi} + \varepsilon^{1/2}\frac{\partial}{\partial \xi}(n_{o+}u_+) = 0 \qquad \cdots\cdots(3b)$$

$$\varepsilon^{3/2}\frac{\partial u_+}{\partial \tau} - \varepsilon^{1/2}\frac{\partial u_+}{\partial \xi} + \varepsilon^{1/2}u_+\frac{\partial u_+}{\partial \xi} + \varepsilon^{1/2}\frac{\partial \phi}{\partial \xi} = 0 \qquad \cdots\cdots(3c)$$

$$\varepsilon^{3/2}\frac{\partial n_{0-}}{\partial \tau} - \varepsilon^{1/2}\frac{\partial n_{0-}}{\partial \xi} + \varepsilon^{1/2}\frac{\partial}{\partial \xi}(n_{0-}u_-) = 0 \qquad \cdots\cdots(3d)$$

$$\varepsilon^{3/2}\frac{\partial u_-}{\partial \tau} - \varepsilon^{1/2}\frac{\partial u_-}{\partial \xi} + \varepsilon^{1/2}u_-\frac{\partial u_-}{\partial \xi} - \varepsilon^{1/2}q\frac{\partial \phi}{\partial \xi} = 0 \qquad \cdots\cdots(3e)$$

$$\varepsilon^{3/2}\frac{\partial n_H}{\partial \tau} - \varepsilon^{1/2}\frac{\partial n_H}{\partial \xi} + \varepsilon^{1/2}\frac{\partial}{\partial \xi}(n_H u_H) = 0 \qquad \cdots\cdots(3f)$$

$$\varepsilon^{3/2}\frac{\partial u_H}{\partial \tau} - \varepsilon^{1/2}\frac{\partial u_H}{\partial \xi} + \varepsilon^{1/2}u_H\frac{\partial u_H}{\partial \xi} - \varepsilon^{1/2}r\frac{\partial \phi}{\partial \xi} = 0 \qquad \cdots\cdots(3g)$$

$$\varepsilon\frac{\partial^2 \phi}{\partial x^2} = \alpha\, n_{0-} - n_{0+} + \beta\left[1 - \frac{T\phi}{\kappa - 3/2}\right]^{-(\kappa - 1/2)} - \sigma\, n_H \qquad \cdots\cdots(3h)$$

where ε measures the size of the perturbed quantities.

We have also used the following power series expansions:

$$n_e = 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \varepsilon^3 n_{e3} + \ldots\ldots \qquad \cdots\cdots(4a)$$

$$n_H = 1 + \varepsilon n_{H1} + \varepsilon^2 n_{H2} + \varepsilon^3 n_{H3} + \ldots\ldots \qquad \cdots\cdots(4b)$$

$$n_{0+} = 1 + \varepsilon n_{0+1} + \varepsilon^2 n_{0+2} + \varepsilon^3 n_{0+3} + \ldots\ldots \qquad \cdots\cdots(4c)$$

$$n_{0-} = 1 + \varepsilon n_{0-1} + \varepsilon^2 n_{0-2} + \varepsilon^3 n_{0-3} + \ldots\ldots \qquad \cdots\cdots(4d)$$

$$u_e = \varepsilon u_{e1} + \varepsilon^2 u_{e2} + \varepsilon^3 u_{e3} + \ldots\ldots \qquad \cdots\cdots(4e)$$

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \ldots\ldots \qquad \cdots\cdots(4f)$$

Decomposition of equations (3a) ~ (3h) in the first order of ε , we get

$$n_{e1} = \left(\frac{\kappa - 1/2}{\kappa - 3/2}\right)T\,\phi_1 \qquad \cdots\cdots(5a)$$

$$\frac{\partial n_{0+1}}{\partial \xi} = \frac{\partial u_{+1}}{\partial \xi} \qquad \cdots\cdots(5b)$$

$$\frac{\partial u_{+1}}{\partial \xi} = \frac{\partial \phi_1}{\partial \xi} \qquad \cdots\cdots\cdots(5c)$$

$$\frac{\partial n_{0-1}}{\partial \xi} = \frac{\partial u_{-1}}{\partial \xi} \qquad \cdots\cdots(5d)$$

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$$\frac{\partial u_{-1}}{\partial \xi} = -q\frac{\partial \phi_1}{\partial \xi} \qquad \cdots\cdots(5e)$$

$$\frac{\partial n_{H1}}{\partial \xi} = \frac{\partial u_{H1}}{\partial \xi} \qquad \cdots\cdots(5f)$$

$$\frac{\partial u_{H1}}{\partial \xi} = r\frac{\partial \phi_1}{\partial \xi} \qquad \cdots\cdots(5g)$$

3.1RESULTS FROM FIRST ORDER CORRECTION

$$n_{0+1} = u_{+1} = \phi_1 \qquad \cdots\cdots\cdots(6a)$$

$$n_{0-1} = u_{-1} = -q\phi_1 \qquad \cdots\cdots\cdots(6b)$$

$$n_{H1} = u_{H1} = r\phi_1 \qquad \cdots\cdots\cdots(6c)$$

$$n_{e1} = \left(\frac{\kappa - 1/2}{\kappa - 3/2}\right) T\,\phi_1 \qquad \cdots\cdots(6d)$$

Decomposition of equations (3a) ~ (3h) in the second order of ε, we get

$$\mathrm{n}_{e2} = \left[\frac{\kappa - 1/2}{\kappa - 3/2}\right] \mathrm{T}\,\varphi_2 + \left[\frac{\kappa^2 - 1/4}{\left(\kappa - 3/2\right)^2}\right] \mathrm{T}^2\,\varphi_1^2 \qquad \cdots\cdots\cdots(7a)$$

$$-\frac{\partial n_{0+2}}{\partial \xi} + \frac{\partial u_{+2}}{\partial \xi} + \frac{\partial}{\partial \xi}\left(n_{0+1}\,u_{+1}\right) + \frac{\partial n_{0+1}}{\partial \tau} = 0 \qquad \cdots\cdots\cdots(7b)$$

$$-\frac{\partial u_{+2}}{\partial \xi} + \frac{\partial \phi_2}{\partial \xi} + u_{+1}\frac{\partial u_{+1}}{\partial \xi} + \frac{\partial u_{+1}}{\partial \tau} = 0 \qquad \cdots\cdots\cdots(7c)$$

$$-\frac{\partial n_{0-2}}{\partial \xi} + \frac{\partial u_{-2}}{\partial \xi} + \frac{\partial}{\partial \xi}\left(n_{0-1}\,u_{-1}\right) + \frac{\partial n_{0-1}}{\partial \tau} = 0 \qquad \cdots\cdots\cdots(7d)$$

$$-\frac{\partial u_{-2}}{\partial \xi} - q\frac{\partial \phi_2}{\partial \xi} + u_{-1}\frac{\partial u_{-1}}{\partial \xi} + \frac{\partial u_{-1}}{\partial \tau} = 0 \qquad \cdots\cdots\cdots(7e)$$

$$-\frac{\partial n_{H2}}{\partial \xi} + \frac{\partial u_{H2}}{\partial \xi} + \frac{\partial}{\partial \xi}\left(n_{H1}\,u_{H1}\right) + \frac{\partial n_{H1}}{\partial \tau} = 0 \qquad \cdots\cdots\cdots(7f)$$

$$-\frac{\partial u_{H2}}{\partial \xi} + r\frac{\partial \phi_2}{\partial \xi} + u_{H1}\frac{\partial u_{H1}}{\partial \xi} + \frac{\partial u_{H1}}{\partial \tau} = 0 \qquad \cdots\cdots\cdots(7g)$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \alpha\, n_{0-2} - n_{0+2} + \beta\left(\frac{\kappa - 1/2}{\kappa - 3/2}\right) T\phi_2 + \beta\left(\frac{\mathrm{K}^2 - 1/4}{\left(\mathrm{K} - 3/2\right)^2}\right) T^2\phi_1^2 - \sigma\, n_{H2} \qquad \cdots\cdots\cdots(7h)$$

Using expressions (2)–(7) in Equations (1a)–(1h) and adopting the reductive perturbation method, we readily obtain the K-dV equation as

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \qquad \cdots\cdots(8a)$$

Where the nonlinearity coefficient,

$$A = B[3(1-\alpha q^2+\sigma r^2)-\beta\left[\frac{\kappa^2-1/4}{(\kappa-3/2)^2}\right]T^2] \quad \cdots\cdots\cdots(8b)$$

and the dispersion coefficient

$$B = \frac{1}{2(1+\alpha q+\sigma r)} \quad \cdots\cdots\cdots(8c)$$

Decomposition of equations (3a) ~ (3h) in the third order of ε, we get

$$-\frac{\partial n_{0+3}}{\partial \xi}+\frac{\partial u_{+3}}{\partial \xi}+\frac{\partial n_{0+2}}{\partial \tau}+\frac{\partial}{\partial \xi}\left(n_{0+1}u_{+2}\right)+\frac{\partial}{\partial \xi}\left(n_{0+2}u_{+1}\right)=0 \quad \cdots\cdots(9a)$$

$$-\frac{\partial u_{+3}}{\partial \xi}+\frac{\partial \phi_3}{\partial \xi}+\frac{\partial u_{+2}}{\partial \tau}+u_{+1}\frac{\partial u_{+2}}{\partial \xi}+u_{+2}\frac{\partial u_{+1}}{\partial \xi}=0 \quad \cdots\cdots(9b)$$

$$n_{e3} = \left(\frac{\kappa-1/2}{\kappa-3/2}\right)T\phi_3+\phi_1\phi_2\left(\frac{\kappa^2-1/4}{(\kappa-3/2)^2}\right)T^2+\phi_1^3\left(\frac{(\kappa^2-1/4)(\kappa+3/2)}{6(\kappa-3/2)^3}\right)T^3 \quad \cdots\cdots(9c)$$

$$-\frac{\partial n_{0-3}}{\partial \xi}+\frac{\partial u_{-3}}{\partial \xi}+\frac{\partial n_{0-2}}{\partial \tau}+\frac{\partial}{\partial \xi}\left(n_{0-1}u_{-2}\right)+\frac{\partial}{\partial \xi}\left(n_{0-2}u_{-1}\right)=0 \quad \cdots\cdots(9d)$$

$$-\frac{\partial u_{-3}}{\partial \xi}-q\frac{\partial \phi_3}{\partial \xi}+\frac{\partial u_{-2}}{\partial \tau}+u_{-1}\frac{\partial u_{-2}}{\partial \xi}+u_{-2}\frac{\partial u_{-1}}{\partial \xi}=0 \quad \cdots\cdots(9e)$$

$$-\frac{\partial n_{H3}}{\partial \xi}+\frac{\partial u_{H3}}{\partial \xi}+\frac{\partial n_{H2}}{\partial \tau}+\frac{\partial}{\partial \xi}\left(n_{H1}u_{H2}\right)+\frac{\partial}{\partial \xi}\left(n_{H2}u_{H1}\right)=0 \quad \cdots\cdots(9f)$$

$$-\frac{\partial u_{H3}}{\partial \xi}+r\frac{\partial \phi_3}{\partial \xi}+\frac{\partial u_{H2}}{\partial \tau}+u_{H1}\frac{\partial u_{H2}}{\partial \xi}+u_{H2}\frac{\partial u_{H1}}{\partial \xi}=0 \quad \cdots\cdots(9g)$$

$$\frac{\partial^2 \phi_2}{\partial \xi^2} = \alpha n_{0-3}-n_{0+3}+\beta T\left[\frac{\kappa-1/2}{\kappa-3/2}\right]\phi_3+\beta T^2\left[\frac{\kappa^2-1/4}{(\kappa-3/2)^2}\right]\phi_1\phi_2+\beta T^3\left[\frac{(\kappa^2-1/4)(\kappa+3/2)}{6(\kappa-3/2)^3}\right]\phi_1^3-\sigma n_{H3} \cdots\cdots(9h)$$

Using expressions (2)–(9) in Equations (1a)–(1h) and adopting the reductive perturbation method, we readily obtain the K-dV equation as

$$\frac{\partial \phi_2}{\partial \tau} + B\frac{\partial^3 \phi_2}{\partial \xi^3} + A\frac{\partial}{\partial \xi}(\phi_1\phi_2) = S(\phi_1) \quad \cdots\cdots(10a)$$

$$\textit{where} \quad S(\phi_1) = -(G_1\phi_1^2\frac{\partial \phi_1}{\partial \xi}+G_2\phi_1\frac{\partial^3 \phi_1}{\partial \xi^3}+G_3\frac{\partial \phi_1}{\partial \xi}\left(\frac{\partial^2 \phi_1}{\partial \xi^2}\right)+G_4\frac{\partial^5 \phi_1}{\partial \xi^5}) \quad \cdots\cdots(10b)$$

Equation (10b) represents the secular term.

$$G_1 = B\ [10A(\alpha q^2-1+\sigma r^2)+3A^2(\alpha q+1+\sigma r)+\frac{15}{2}\left(\alpha q^3+1+\sigma r^3\right)-\beta T^3\left(\frac{(\kappa^2-1/4)(\kappa-1/2)}{2(\kappa-3/2)^3}\right)] \quad \cdots\cdots(10c)$$

$$G_2 = 16B^2(\alpha q^2-1-\sigma r^2)+6AB^2(\alpha q+1+\sigma r) \quad \cdots\cdots(10d)$$

$$G_3 = 4B^2(\alpha q^2-1-\sigma r^2)+3AB^2(\alpha q+1+\sigma r) \quad \cdots\cdots(10f)$$

$$G_4 = \frac{3B^2}{2} \quad \cdots\cdots(10g)$$

Results and Discussion

We have used the parameters observed at the coma of comet Halley [Brinca and Tsurutani, 1987] to study the soliton solution of first order K-dV equation and K-dV equation with secular term. The densities of hydrogen ion observed at comet Halley was 4.95 cm^{-3}, i.e., nH= 4.95 $cm^{-3.}$ The temperature of these ions was $T_H = 1.16 \times 10^4 K$. The solar electron temperature was taken as the temperature of superthermal electron i.e., $T_e = 2 \times 10^5 K$. The spacecraft Giotto observed negatively charged ions in the mass peaks of 7-19, 22-65 and 85-110 amu in the inner coma of comet Halley: which indicates the presence of negatively charged oxygen ions unambiguously [Chaizy et al., 1991]. The equilibrium densities of negatively and positively charged oxygen ion is given as, $n_{o-} = 0.05 cm^{-3}$ and $n_{o+} = 0.5 cm^{-3}$ respectively[Brinca and Tsurutani, 1987; Chaizy et al., 1991].

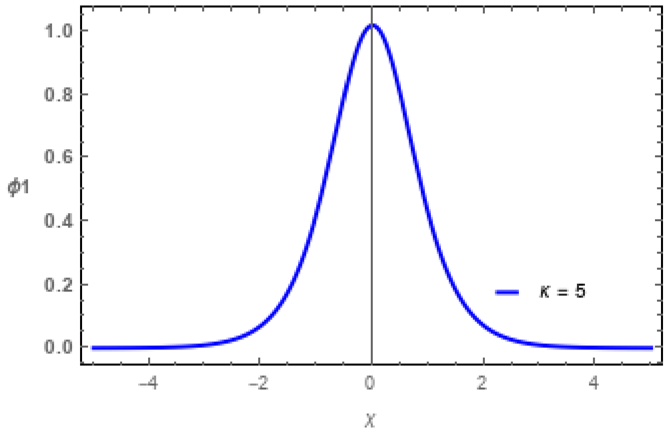

Fig. .1: variation of soliton solution ϕ 1 versus space variable 'X'.

Figure 1 is a plot of soliton solution versus space variable 'X'for different spectral indices. We get solitons with its amplitude and width increases with increasing spectral indices. The other parameters used are given below,

u = 0.5, $n_{o-} = 0.05\, cm^{-3}$, $n_{o+} = 0.5\, cm^{-3}$, $nH = 4.95\, cm^{-3}$, $T_e = 2 \times 10^5 K$,

$T_H = 1.16 \times 10^4 K$, $\kappa = 3$, $m_+ = m_- = 1.67 \times 10^{-24} gms$, $m_H = 1.67 \times 10^{-24} gms$,

$and\ m_e = 9 \times 10^{-28} gms.$

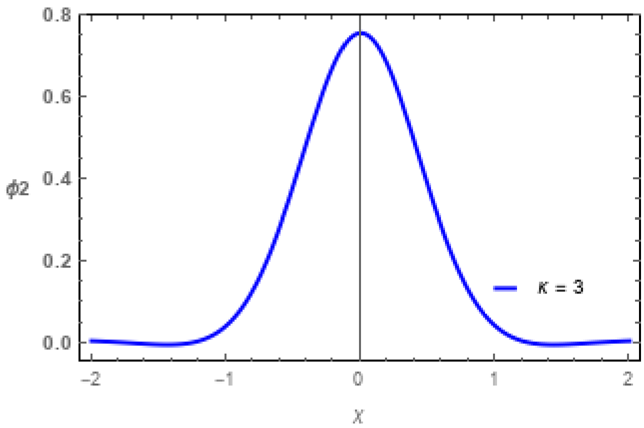

Fig. 2: variation of solitonsolution ϕ 2 versus space variable X.

Figure 2 is a plot of second order soliton solution versus space variable 'X' for different spectral indices. We get solitons with its amplitude and width increases with increasing spectral indices. The other parameters used are given below,

$u = 0.5,\ n_{o-} = 0.05\ cm^{-3},\ n_{o+} = 0.5\ cm^{-3},\ nH = 4.95\ cm^{-3},\ T_e = 2\times10^5 K,$

$T_H = 1.16\times10^4 K,\ \kappa = 3,\ m_+ = m_- = 1.67\times10^{-24} gms,\ m_H = 1.67\times10^{-24} gms,$

$and\ m_e = 9\times10^{-28} gms.$

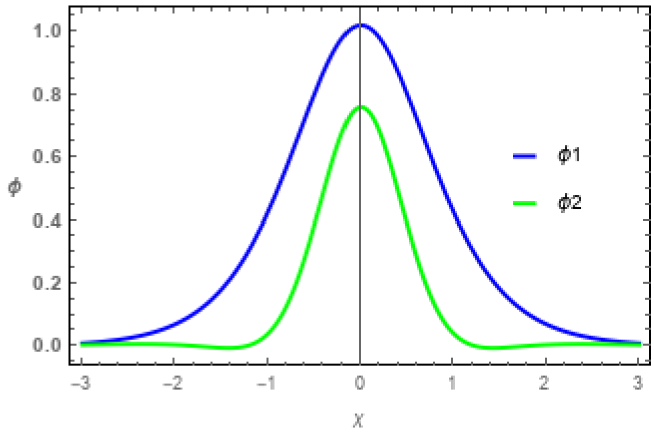

Fig.3; comparison of the two soliton solutions $\phi1$ and $\phi2$ versus space variable X.

Figure 3 is a plot of comparison between the two soliton solutions $\phi1$ and $\phi2$ .versus space variable 'X'. Here the amplitude of $\phi1$ is greater than that of $\phi2$. This curve is particularly important because in the second order K-dV equation, coefficient A represents nonlinearity and B represents dispersion of the wave.It is seen that A is always negative and its magnitude increases with increasing specral index. The amplitude is representative of nonlinearity. The other parameters are similar to that of figure 1.

Figure 4 is a plot of variation of the secular term of second order K-dV equation versus the space variable 'X'. It can be seen that the secular term is negative and is inversely proportional to the second order soliton solution $\phi2$. As $\phi2$ increases, S($\phi1$)decreases andvice versa. The other parameters are similar to that of figure 1.

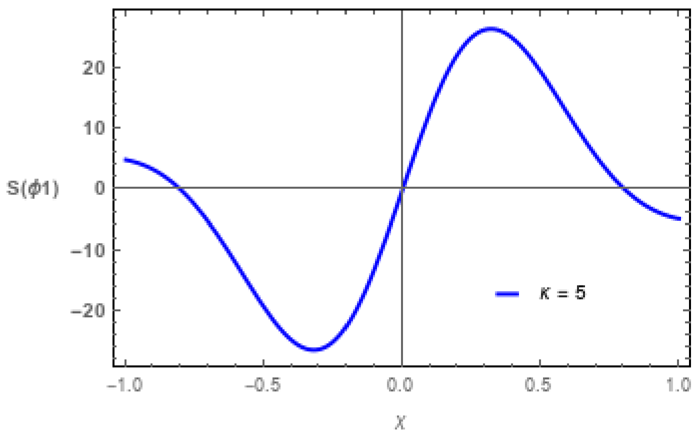

Fig.4; variation of secular term of second order K-dV equation S($\phi1$) versus space variable X.

Conclusion

We have studied solitary waves in a four component plasma consisting of superthermal electrons, hydrogen ions and positively and negatively charged oxygen ions. We have seenthat the amplitude and width of the soliton increases with increasing spectral indices. On comparing the first and second order soliton solutions, we find that the amplitude and width of first order soliton is greater than that of second order soliton. This is because the coefficient A in first order K-dV equation represents nonlinearity and B represents dispersion of the wave.It is seen that A is always negative and its magnitude increases with increasing spectral index. The amplitude is a representative of nonlinearity. Because of having lower nonlinearity than the first order soliton, the second order soliton has smaller amplitude than the first. We have also find that the secular term of the second order K-dV equation is negative. It is inversely proportional to the second order soliton solution.

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